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SOME EXACT SOLUTIONS OF THE PROBLEM OF THE HYPERSONIC OR SUPERSONIC FLOW OF A GAS PAST A SLIPPING WING WITH STALL FENCE

A. I. Golubinskiy and A. N. Ivanov

Translation of "Nekotoryye tochnyye resheniya zadachi obtekaniya skol'zyashchego kryla s peregorodkoy sverkhzvukovym i giperzvukovym potokom gaza"
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A. I. Golubinskiy and A. N. Ivanov

Construction of a class of three-dimensional configurations representing a combination of a slipping or delta wing and a stall fence mounted on the wing parallel to the oncoming flow. It is shown that for such configurations, exact solutions for hypersonic or supersonic flow can be obtained without recourse to the small perturbation method. A solution for the flow past two intersecting yawing wings is obtained as a special case. Flow analysis reveals strong interference at high speeds leading to much higher local pressures than in the case of an isolated wing.

Individual examples of three-dimensional bodies are known for which it is possible to solve the flow problem, for example, bodies of star shaped (polygons) cross section (refs. 1, 2) or bodies of revolution (cone, ellipsoid) at an angle of attack, etc. The role of such individual solutions is significant for the clarification of characteristic properties associated with the three-dimensional flow around bodies. /145*

The application of the hypersonic method of small perturbations (ref. 3) which reduces the problem of three-dimensional flow to the problem of the nonstationary gas flow in two dimensions, and also the utilization of one known family of exact solutions of the plane problem concerning nonstationary motion (ref. 4) make it possible to investigate the flow around a slipping wing with an end fence at hypersonic velocities in some particular cases.

The present work constructs a wider class of three-dimensional bodies of this type (the combination of a slipping or delta wing with an end fence, directed parallel to the incident flow), for which it is possible to construct simple exact solutions of the supersonic and hypersonic flow problem without applying the method of small perturbations. In a particular case these solutions are described by flow around 2 mutually intersecting slipping wings.

An example of flow around these bodies shows an interesting phenomena of strong interference at large velocities when such interference produces local pressures, which are considerable, in excess of pressures on the isolated wing.

1. Let us consider the flow over the region of a delta (or sweptback) wing similar to the one shown in figure 1 at a supersonic velocity directed towards the axis of the coordinates x.

*Numbers given in margin indicate pagination in original foreign text.

A plane fence ODF is installed at the end of the wing, which is parallel to the incident flow and inclined at an angle ξ with respect to the y axis. A profile of the wing has the shape of a wedge with an angle δ (along the normal to the leading edge). The angle of the leading edge OA with respect to the direction of the incident flow is designated by α.

We shall attempt to find those geometric relationships for the parameters α, δ, ξ and those Mach numbers of the flow for which the following simple flow picture can exist for the wing: the compression shock AOD, attached to the leading edge falls on the fence DOF and after reflection from this fence falls on the upper surface of the wing at a right angle. Then the new reflection of the shock does not occur and the entire flow picture can be computed by means of known relationships for oblique compression shocks.

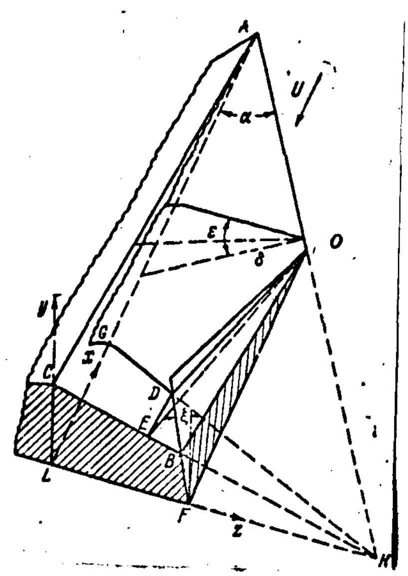

Figure 1.

We designate the normal flow component at the leading edge of the wing by

$$M_1 = M_0 \sin \alpha \tag{1.1}$$

For the compression shock which is attached to the wing, we have (ref. 5)

$$\operatorname{ctg} \delta = \left[\frac{\gamma + 1}{2} \frac{M_1^2}{M_1^2 \sin^2 \varepsilon - 1} - 1\right] \operatorname{tg} \varepsilon \tag{1.2}$$

were ε is the angle between the attached shock and the plane xz, γ is the ratio of the specific heats of the gas.

We designate by ω_1 and ω_2 respectively the angles between the incident and the reflected shocks and the plane of the fence. It is easy to see that /146

$$\omega_1 = \arccos (\cos \varepsilon \sin \xi + \sin \varepsilon \cos \xi \cos \alpha) \tag{1.3}$$

The angle ω_2 is expressed in terms of ω_1 and the intensity of the incident shock η, which is equal to the ratio of the pressures p_2/p_1 at the shock, in the following manner (ref. 6):

$$\tau_2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}, \quad \tau_1 = \operatorname{ctg}\omega_1, \quad \tau_2 = \operatorname{ctg}\omega_2$$
$$A = s\,[(\nu - 1)\,s - (\nu - 2)]\,(\tau_1^2 + 1) - 1, \quad \nu = (\gamma + 1)/(\gamma - 1)$$
$$B = -(\nu - 1)\,\tau_1\,[s^2\,(\tau_1^2 + 1) - 1], \quad C = \nu\,[s\,(\tau_1^2 + 1) - 1] \qquad (1.4)$$
$$s = \frac{\nu\eta + 1}{(\nu - 1)(\eta - 1)}, \quad \eta = \frac{2\gamma}{\gamma + 1}\,M_1^2 \sin^2 \varepsilon - \frac{1}{\nu}$$

It should be pointed out that the "plus" in front of the root in equation (1.4) corresponds to a weak reflection while a "minus" corresponds to a strong reflection. Finally from the condition that the reflected shock is not perpendicular to the upper wing surface, and also from the fact that the planes of the incident and reflected shocks belong to the same beam of planes (passing through the line OD) we obtain

$$\cos(\varepsilon - \delta) = (\tau_1 + \tau_2)\sin\omega_1\,(\cos\delta \sin\xi + \sin\delta\cos\xi\cos\alpha) \qquad (1.5)$$

Equations (1.2)-(1.5) give us a relationship associating 7 parameters M_0, α, δ, ϵ, ξ, ω_1, ω_2. Therefore three parameters can be assigned independently; for example, the geometry of the body (α and δ) and the number M_0. The fourth parameter ξ (the slope of the fence) depends on these parameters. We note that it is impossible to solve the above system of equations in an explicit form except for the case considered below in Section 3.

2. The region where the considered flows exist may be determined from the following physical considerations.

(a) The angle of the leading edge must satisfy the inequility

$$\alpha > \arcsin\frac{1}{M_0} \qquad (2.1)$$

(b) The wedge angle δ must be less than some limiting value δ_*, determined from equation (1.2) when

$$\sin^2\varepsilon_* = \frac{1}{\gamma M_1^2}\left({}^1\!/_4\,(\gamma + 1)\,M_1^2 - 1 + \sqrt{(\gamma + 1)\,[1 + {}^1\!/_2\,(\gamma - 1)M_1^2 + {}^1\!/_{16}\,(\gamma + 1)M_1^4]}\right) \qquad (2.2)$$

(c) In order for a proper reflection to exist it is necessary that the discriminant in equation (1.4) be $B^2 - 4AC \geq 0$, which corresponds to $\omega_1 \leq \omega_{**}$, where ω_{**} depends on p_2/p_1 and γ.

(d) From our consideration we do not exclude the case of strong shock reflection although the realization of strong reflection in practice is doubted by some investigators (refs. 6 and 7).

Let us assume however that strong reflection of slipping shocks from the fence may be realized under the condition when the total velocity behind the reflected shock will be supersonic, i.e., when $M_3 > 1$.

Since it is impossible to solve the equations in explicit form, the boundaries of the existing region corresponding to conditions (c) and (d), may be found only by numerical calculations.

3. It is known that the equation (1.4) which associates the angles of incidents and reflection of the shock, has one characteristic solution when these angles are the same and when they do not depend on the ratio of pressures but are equal (ref. 6)

$$\omega_1 = \omega_2 = \omega^* = \operatorname{arc\,ctg}\left(\frac{\gamma+1}{3-\gamma}\right)^{1/2} \qquad (3.1)$$

In this case the symmetric reflection of the pressure drop on the oblique reflected shock is the same as in the case of direct reflection and the solution depends only on two parameters because for the five unknowns (M_0, α, δ, ϵ, ξ) there are three equations (1.2), (1.3), (1.5). For this case it is possible to solve the system in explicit form if M_1 and ϵ are assigned as the independent parameters. Then δ is determined from (1.2) and we also have

$$\sin\xi = \frac{\sin\varepsilon\cos(\varepsilon-\delta) - \frac{1}{2}(\gamma+1)\sin\delta}{\sqrt{\gamma+1}\,\sin(\varepsilon-\delta)}, \quad \cos\alpha = \frac{\sqrt{\gamma+1}}{2\sin\varepsilon\cos\xi} - \operatorname{ctg}\varepsilon\operatorname{tg}\xi \qquad (3.2)$$

As an illustration figure 2 presents the function $\xi = \xi(\alpha)$ for various M_0 /147
in the case when $\gamma = 1.4$ (the angles ξ, α are given in degrees).

4. The region where the solution exists in the symmetric case for the independent variables M_1 and ξ can be easily determined from conditions (a), (b), (d) of section 2 and also from the condition

$$\epsilon \geq \operatorname{arc\,sin}\frac{1}{M_1}.$$

As a result we obtain

$$\operatorname{arc\,sin}\frac{1}{M_1} \leqslant \varepsilon \leqslant \varepsilon_* \qquad (4.1)$$

where ϵ_* is determined by equation (2.2), and where we also have $\epsilon \geq \mathrm{arc\ tg}\, k_+$ or $\epsilon \leq \mathrm{arc\ tg}\, k_-$, where

$$k_\pm = \frac{1}{(\gamma - 1) M_1^2 + 2} \left\{ \frac{(\gamma - 1)\sqrt{(\gamma + 1)}}{2\sqrt{3 - \gamma}} M_1^2 \pm \right. \\ \left. \pm \left[\frac{(\gamma - 1)^2 (\gamma + 1)}{4(3 - \gamma)} M_1^4 - 2(\gamma - 1) M_1^2 - 4 \right]^{1/2} \right\} \qquad (4.2)$$

The regions where solutions exist for the case $\gamma = 1.4$ and $\gamma = 1.67$, constructed from these conditions, are shown in figure 3. The lines AB and BD correspond to the condition (4.1), while the line DE corresponds to condition (4.2). The line FG divides the region of weak (to the left), and strong reflections for $\gamma = 1.4$). It is obtained from the condition (ref. 7)

$$p_2 / p_1 = 7.02 \qquad (4.3)$$

It should be pointed out that according to the calculations, condition (d) is satisfied automatically within the limits of the specified region of existence.

5. If the angle of the sweptback wing is $\alpha \to 0$ while $M_0 \to \infty$, but such that $M_1 = M_0\, \alpha > 1$, then equations (1.3) and (1.5) are simplified and assume the form

$$\omega_1 = \tfrac{1}{2}\pi - \epsilon - \xi$$
$$\omega_2 = \delta + \xi$$

The resulting system of equations coincides completely with the system of equations which describe the flow around a wedge that penetrates a stationary inclined fence (fig. 4). Thus in this case the known law of plane cross section is confirmed.

During the specified transition the number of characteristic parameters is reduced to two (M_1, δ) but it is nevertheless impossible to obtain the solution in the explicit form. However when M_1 are sufficiently large so that $M_1^2 \sin^2 \epsilon \gg 1$ (i.e. when we can neglect the initial pressure p_1), the solution is sub- /148
stantially simplified. Indeed when

$$\delta = \omega^* = \mathrm{arc\ ctg} \left(\frac{\gamma + 1}{3 - \gamma} \right)^{\frac{1}{2}}$$

we obtain

$$\epsilon = \tfrac{1}{2}\pi - \omega^*, \quad \xi = 0, \quad \omega_1 = \omega_2 = \omega^*$$

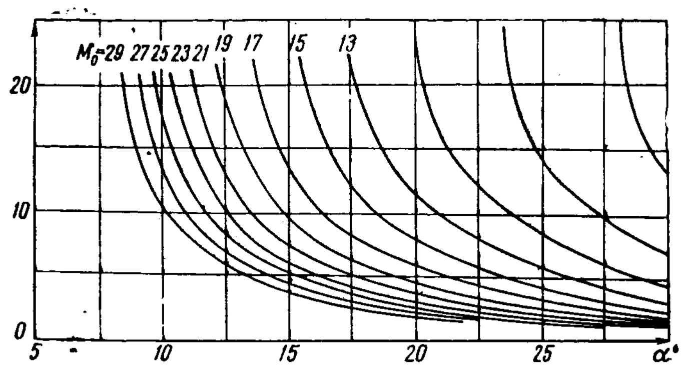

Figure 2

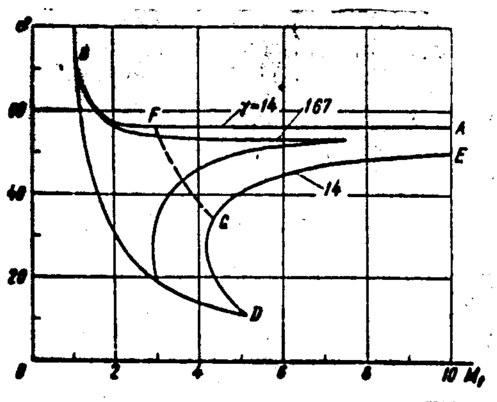

Figure 3

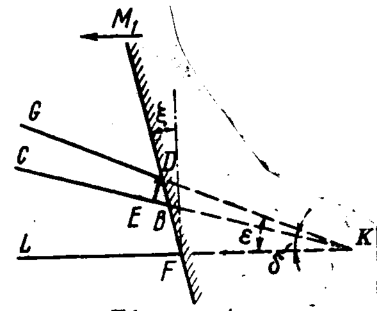

Figure 4

which corresponds to the symmetric case. When $\delta \neq \omega^*$ we can obtain the solution in the form of an expansion with respect to the symmetric case in powers of ξ, it is only necessary to write the relationship for the incident and reflected shock in the form given in reference 7.

For the first two terms of the expansion we have

$$\left.\begin{array}{c} \delta = \omega^* - \dfrac{(2-\gamma)(5\gamma+1)}{4\gamma(3-\gamma)}\xi, \quad \varepsilon = {}^1/_2\pi - \omega^* - \dfrac{5\gamma+1}{4\gamma(3-\gamma)}\xi \\ \omega_1 = \omega^* + \dfrac{4\gamma^2-7\gamma+1}{4\gamma(3-\gamma)}\xi, \quad \omega_2 = \omega^* + \dfrac{\gamma^2+3\gamma-2}{4\gamma(3-\gamma)}\xi \\ \eta_3 = \dfrac{p_3}{p_2} = \dfrac{3\gamma-1}{\gamma-1} + \dfrac{2\gamma}{\gamma-1}\left(\dfrac{3-\gamma}{\gamma+1}\right)^{1/2}\xi \end{array}\right\}$$

where p_3/p_2 is the ratio of pressures at the reflected shock.

It is of interest to compare the lift force Y_1 for the BE region of the wing (fig. 4), which adjoins the fence (taking into account the force acting on the fence), with the lift force Y_2 for the KB region of the wing when the fence is absent. It is easy to see that

$$\frac{Y_1}{Y_2} = \frac{p_3 \operatorname{tg}(\varepsilon - \delta)}{p_2 \cos(\delta + \varepsilon)} \left[\sin(\delta + \varepsilon) - \frac{\sin \xi}{\cos \delta} \right] \qquad (5.1)$$

As a result of this, expansion in terms of ξ gives us

$$\frac{Y_1}{Y_2} = \frac{3\gamma - 1}{\gamma + 1} + \frac{1}{\gamma + 1} \left[2\gamma - \frac{(3\gamma - 1)(5\gamma + 1)}{\gamma(3 - \gamma)} \right] \left(\frac{3 - \gamma}{\gamma + 1} \right)^{1/2} \xi$$

Let us consider this solution for the case $\xi = 0$ when $\gamma \to 1$. In this case the region of increased pressure is reduced while the pressure itself increases in such a way that $Y_1/Y_2 \to 1$. Thus when $\xi = 0$, $M_1 \to \infty$ and $\gamma \to 1$, a concentrated lift force occurs at the point where the wing is joined with the fence and this force is equal to the force which act on the cutoff section of the wing.

We note that the occurence of a substantial concentrated lift force at the point where the wing joins the fence is difficult to explain within the framework of known Newton's theory for hypersonic flow around bodies and, apparently, is directly associated with the presence of the strong reflection of the head shock from the fence. Therefore it is of prime interest to investigate experimentally the established flows at high supersonic velocities. We note that when $\gamma > 1$ the additional forces produced by the end fence become compatible with forces acting on part of the wing cut off by the fence (fig. 5).

We should point out that during hypersonic transition the flow behind the reflected shock wave remains supersonic. Indeed when $M_1 \to \infty$ and $\xi = 0$ we have

$$M_3^2 = \frac{2}{\gamma - 1} \left[\frac{\gamma + 1}{(3\gamma - 1)\sin^2\alpha} - 1 \right]$$

i.e., in the considered case when $\alpha \to 0$ we obtain $M_3 \to \infty$. Thus during the hypersonic transition we may assume that the strong reflection of the head shock from the fence is realized.

6. Let us evaluate the drag forces which act on the section of the wing with the fence, in a hypersonic flow.

We note that when $\alpha \to 0$ the wave drag X_w also tends to zero, while the drag of the body is determined by the force of friction. In this connection we com- /149
pare the friction force X_{f1}, acting on the section of the wing with the fence (including the friction force acting on the fence), with the friction force X_{f2} acting on the cut-off part of the wing.

To evaluate the forces of friction we use the approximate equation proposed by V. A. Dashkin, according to which the friction force is given by the following expression when the boundary layer for the plate is laminar, when $M_1 \gg 1$,

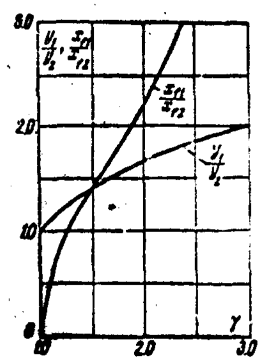

Figure 5

when there is strong heat exchange on the surface of the body, and also when the relationship between the coefficient of viscosity as a function of temperature $\mu \sim T^{\omega}$ follows an exponential law:

$$X_f = c_f F\,{}^1\!/_2\,\rho u^2 = 1.33\,\sqrt{\lambda / Rl}\,F\,{}^1\!/_2\,\rho u^2, \qquad \lambda \approx [(\gamma - 1)\,M_1^2 f\,(P)]^{\omega - 1} \quad (6.1)$$

Here P is the Prandtl number, F is the area of the plate, ρ is the density, u is the flow velocity at the external boundary of the boundary layer, R is the Reynolds number when the length of the plate is equal to l.

Substituting the expressions for all of the variables into equation (6.1) we obtain

$$\frac{X_{f1}}{X_{f2}} = \left(\frac{p_3}{p_2}\right)^{1/2} \operatorname{tg}(\varepsilon - \delta)\,\frac{1 + \sin\omega_2}{\cos\omega_2}$$

which, in an expanded form, gives us

$$\frac{X_{f1}}{X_{f2}} = \frac{\sqrt{(3\gamma - 1)(\gamma - 1)}}{\gamma + 1}\left(1 + \frac{2}{\sqrt{3 - \gamma}}\right) + \\ + \frac{1}{\nu^{1/2}}\left[\frac{2\gamma}{\gamma + 1}\left(\frac{3 - \gamma}{(3\gamma - 1)(\gamma + 1)}\right)^{1/2} - \frac{1}{\gamma}\left(\frac{3\gamma - 1}{3 - \gamma}\right)^{1/2} - \frac{\gamma^2 - \gamma + 10}{2\gamma(3 - \gamma)^2}\sqrt{3\gamma - 1}\right]\xi$$

The variation in X_{fs}/X_{fs} as a function of γ when $\xi = 0$ is shown in figure 5.

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SOME EXACT SOLUTIONS OF THE PROBLEM OF THE HYPERSONIC OR SUPERSONIC FLOW OF A GAS PAST A YAWING WING WITH STALL FENCE [NEKOTORYE TOCHNYE RESHENIIA ZADACHI OBTEKANIIA SKOL'ZIASHCHEGO KRYLA S PEREGORODKOI SVERKHZVUKOVYM I GIPERZVUKOVYM POTOKOM GAZA]

A. I. Golubinskii and A. N. Ivanov.

Akademiia Nauk SSSR, Izvestiia, Mekhanika Zhidkosti i Gaza, Jan.-Feb. 1966, p. [illegible]-149. 7 refs. In Russian.

Construction of a class of three-dimensional configurations representing a combination of a yawing or delta wing and stall fence mounted on the wing parallel to the oncoming flow. It is shown that for such configurations, exact solutions for hypersonic or supersonic flow can be obtained without recourse to the small perturbation method. A solution for the flow past two intersecting yawing wings is obtained as a special case. Flow analysis reveals strong interference at high speeds leading to much higher local pressures than in the case of an isolated wing. V.P.

НЕКОТОРЫЕ ТОЧНЫЕ РЕШЕНИЯ ЗАДАЧИ ОБТЕКАНИЯ СКОЛЬЗЯЩЕГО КРЫЛА С ПЕРЕГОРОДКОЙ СВЕРХЗВУКОВЫМ И ГИПЕРЗВУКОВЫМ ПОТОКОМ ГАЗА

А. И. ГОЛУБИНСКИЙ, А. Н. ИВАНОВ
(Москва)

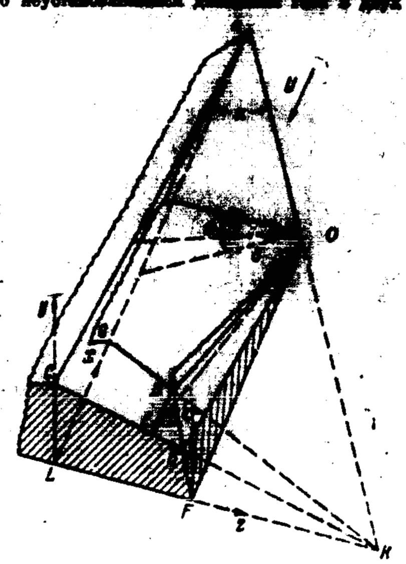
Известные [illegible] пространственной формы, для которых [illegible] [illegible] о неустановившихся [illegible]

ного [illegible]

[illegible]

[illegible] этих тел [illegible] больших [illegible] давления, [illegible] превосходящие давления на [illegible] крыле.

1. Рассмотрим обтекание участка треугольного (или стреловидного) крыла, [illegible] фиг. 1, сверхзвуковым потоком газа, направленным навстречу оси координат x.



Фиг. 1

На [illegible] крыла установлена плоская перегородка ODF, параллельная набегающему потоку и наклоненная под углом ξ к оси y. Профиль крыла имеет форму клина с углом δ (по нормали к передней кромке). Угол передней кромки OA относительно направления набегающего потока обозначим через α.

Попытаемся найти такие соотношения геометрических параметров α, δ, ξ и числа Маха M_0 потока, при которых на крыле может иметь место следующая простейшая картина обтекания: скачок уплотнения AOD, присоединенный к передней кромке, падает на перегородку DOF и, отразившись от нее, падает на [illegible] поверхность крыла под прямым углом. Тогда нового отражения скачка не происходит, и всю картину течения легко рассчитать до конца при помощи известных соотношений для косых скачков уплотнения.

Обозначим нормальную к передней кромке крыла составляющую потока через

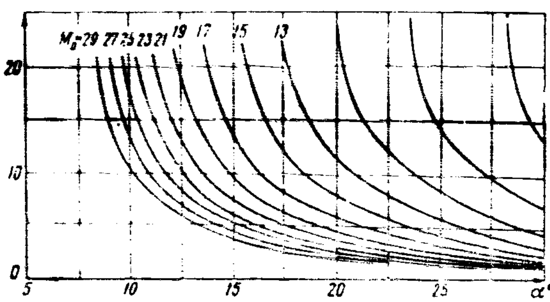
$$[illegible] \tag{1.1}$$

Для присоединенного [illegible] уплотнения имеем [*]

$$[illegible] \frac{M_1^2}{M_1^2 \sin^2 \varepsilon - 1} - 1 \Big] \operatorname{tg} \varepsilon \tag{1.2}$$

где ε — угол между присоединенным скачком и плоскостью xz, γ — отношение теплоемкостей газа.

На фиг. 2 в качестве иллюстрации приведены зависимости $\xi = \xi(\alpha)$ при различных M_0 для случая $\gamma = 1.4$ (углы ξ, α в градусах).



Фиг. 2

4. Область существования решения в симметричном случае при независимых переменных M_1 и ε легко определить из условий (а), (б), (г) п. 2, а также из условия

$$\varepsilon \geqslant \arcsin \frac{1}{M_1}$$

В результате получим

$$\arcsin \frac{1}{M_1} \leqslant \varepsilon \leqslant \varepsilon_* \tag{4.1}$$

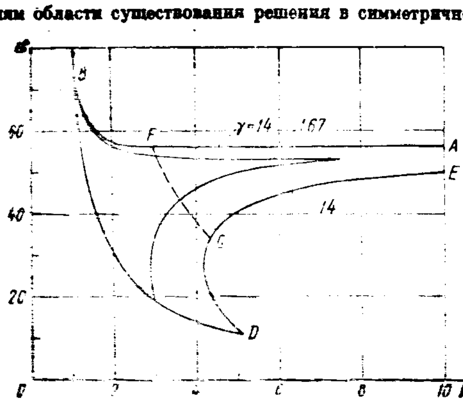
где ε_* определяется уравнением (2.2), а также $\varepsilon \geqslant \operatorname{arc\,tg} k_+$ или $\varepsilon \leqslant \operatorname{arc\,tg} k_-$, где

$$k_\pm = \frac{1}{(\gamma-1) M_1^2 + 2} \left\{ \frac{(\gamma-1)\sqrt{(\gamma+1)}}{2\sqrt{3-\gamma}} M_1^2 \pm \right.$$
$$\left. \pm \left[\frac{(\gamma-1)^2(\gamma+1)}{4(3-\gamma)} M_1^4 - 2(\gamma-1) M_1^2 - 4 \right]^{1/2} \right\} \tag{4.2}$$

Построенные по этим условиям области существования решения в симметричном случае для $\gamma = 1.4$ и $\gamma = 1.67$ приведены на фиг. 3. Линии AB и BD соответствуют условию (4.1), линия DE — условию (4.2). Линия FG разделяет область слабого (слева) и сильного отражений для $\gamma = 1.4$. Она находится из условия [?]

$$p_2 / p_1 = 7.02 \tag{4.3}$$

Следует отметить, что, согласно расчетам, в пределах указанной области существования условие (г) выполняется автоматически.



Фиг. 3

5. Если угол стреловидности $\alpha \to 0$, а $M_0 \to \infty$, но так, что $M_1 = M_0 \alpha > 1$, то уравнения (1.2) и [illegible] упрощаются и приобретают вид

$$\omega_1 = \tfrac{1}{2}\pi \qquad \varepsilon = \xi$$
$$\omega_2 = \delta \qquad \xi$$

Получившаяся в результате система уравнений полностью совпадает с системой уравнений, [illegible] обтекание клина, пронизывающего [illegible]ную наклонную стенку (фиг. 4). [illegible] в данном случае подтверждается известный закон [illegible].

При [illegible] определяющих [illegible] сокращается до двух (M_1, ε), [illegible] получить решение [illegible] в явном виде. Однако при достаточно больших M_1, таких, что $M_1^2 \sin^2 \varepsilon \gg 1$ (т. е. когда можно пренебречь начальным давлением p_1), решение существенно упрощается. Действительно, при

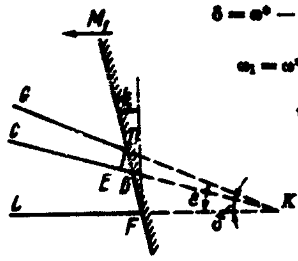
$$\delta = \omega^\circ = \operatorname{arc\,ctg}\left(\frac{\gamma+1}{3-\gamma}\right)^{1/2}$$

получаем

$$\varepsilon = {}^1/_2\pi - \omega^\circ, \quad \xi = 0, \quad \omega_1 = \omega_2 = \omega^\circ$$

что соответствует симметричному случаю. Для $\delta \neq \omega^\circ$ можно получить решение в виде разложения относительно симметричного случая по степеням ξ, необходимо только записать соотношения для падающего и отраженного скачка в форме [7].

Для первых двух членов разложения имеем



Фиг. 4

$$\delta = \omega^\circ - \frac{(2-\gamma)(5\gamma+1)}{4\gamma(3-\gamma)}\,\xi, \quad \varepsilon = {}^1/_2\pi - \omega^\circ - \frac{5\gamma+1}{4\gamma(3-\gamma)}\,\xi$$

$$\omega_1 = \omega^\circ + \frac{4\gamma^2-7\gamma+1}{4\gamma(3-\gamma)}\,\xi, \quad \omega_2 = \omega^\circ + \frac{\gamma^2+3\gamma-2}{4\gamma(3-\gamma)}\,\xi$$

$$\eta_3 = \frac{p_3}{p_2} = \frac{3\gamma-1}{\gamma-1} + \frac{2\gamma}{\gamma-1}\left(\frac{3-\gamma}{\gamma+1}\right)^{1/2}\xi$$

где p_3 / p_2 — отношение давлений на отраженном скачке.

Представляет интерес сравнить подъемную силу Y_1 для участка крыла BE (фиг. 4), примыкающего к перегородке (учитывая и силы, действующие на перегородку), с подъемной силой Y_2 для участка крыла KB при отсутствии перегородки. Нетрудно видеть, что

$$\frac{Y_1}{Y_2} = \frac{p_3 \operatorname{tg}(\varepsilon-\delta)}{p_2 \cos(\delta+\varepsilon)}\left[\sin(\delta+\varepsilon) - \frac{\sin\xi}{\cos\delta}\right] \qquad (5.1)$$

Это при разложении по ξ дает

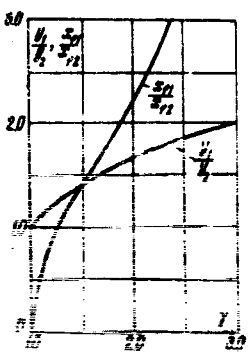
$$\frac{Y_1}{Y_2} = \frac{3\gamma-1}{\gamma+1} + \frac{1}{\gamma+1}\left[2\gamma - \frac{(3\gamma-1)(5\gamma+1)}{\gamma(3-\gamma)}\right]\left(\frac{3-\gamma}{\gamma+1}\right)^{1/2}\xi$$

Рассмотрим это решение при $\xi = 0$, когда $\gamma \to 1$. В этом случае область повышенного давления сужается, а само давление растет так, что $Y_1 / Y_2 \to 1$. Таким образом, при $\xi = 0$, $M_1 \to \infty$ и $\gamma \to 1$ в месте сопряжения крыла с перегородкой возникает сосредоточенная подъемная сила, равная силе, действующей на отсеченный участок крыла.

Отметим, что появление значительной сосредоточенной подъемной силы в месте сопряжения крыла с перегородкой трудно объяснимо в рамках известной ньютоновской теории для гиперзвукового обтекания тел и, по-видимому, непосредственно связано с наличием сильного [illegible] скачка от перегородки. Поэтому представляет [illegible] интерес [illegible] найденных течений при больших сверхзвуковых скоростях. Заметим, что при значениях $\gamma > 1$ [illegible] силы, создаваемые [illegible] соизмеримы с силами, действующими на отсеченную перегородкой часть крыла (фиг. 5).

[illegible] потери на отраженной ударной [illegible] Действительно, при $M_1 \to \infty$ для $\xi = 0$ [illegible]

$$[illegible]$$



Фиг. 5

[illegible]

трения X_{f1}, действующую на участок крыла с перегородкой (включая и силу трения, действующую на перегородку), с силой трения X_{f2}, действующей на отсеченную часть крыла.

Для оценки сил трения воспользуемся приближенной формулой, предложенной В. А. [illegible], согласно которой для пластины при ламинарном пограничном слое, $M_1 \gg 1$ и сильном теплообмене на поверхности тела, а также при степенной зависимости коэффициента вязкости от температуры $\mu \sim T^{\omega}$ сила трения равна

$$X_f = c_f F \tfrac{1}{2} \rho u^2 = 1.33 \sqrt{\lambda / Rl}\, F \tfrac{1}{2} \rho u^2, \qquad \lambda \approx [(\gamma - 1) M_1^2 / (P)]^{\omega - 1} \qquad (6.1)$$

Здесь P — число Прандтля, F — площадь пластины, ρ — плотность, u — скорость потока на внешней границе пограничного слоя, R — число Рейнольдса при длине пластины, равной l.

Подставляя в уравнение (6.1) выражения для всех входящих в него переменных, нетрудно получить

$$\frac{X_{f1}}{X_{f2}} = \left(\frac{p_2}{p_1}\right)^{1/2} \operatorname{tg}(\varepsilon - \delta) \frac{1 + \sin \omega_1}{\cos \omega_1}$$

что в разложении дает

$$\frac{X_{f1}}{X_{f2}} = \frac{\sqrt{(2\gamma - 1)(\gamma - 1)}}{\gamma + 1}\left(1 + \frac{2}{\sqrt{3 - \gamma}}\right) +$$

$$+ \frac{1}{\sqrt{2}}\left[\frac{2\gamma}{\gamma + 1}\left(\frac{3 - \gamma}{(2\gamma - 1)(\gamma + 1)}\right)^{1/2} - \frac{1}{\gamma}\left(\frac{2\gamma - 1}{3 - \gamma}\right)^{1/2} - \frac{\gamma^2 - \gamma + 10}{2\gamma (3 - \gamma)^2} \sqrt{2\gamma - 1}\right]\xi$$

Зависимость X_{f1} / X_{f2} от γ при $\xi = 0$ приведена на фиг. 5.

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